THE INFLUENCE OF RESIDUAL STRESSES ON THE ACCURACY MACHINING ON MACHINE TOOLS

M.F. KERIMZHANOVA, Aiman Z. GABDULLINA

Abstract
The article discusses theoretical basic relations for described deformations of thin cylindrical shells from the residual stresses arising after machining. The paper presents the usage of energy stability theory of elastic shells.

Keywords
stress, strain, accuracy, tolerances, fit, size chain

In the process of machining work pieces on machine tools, in the surface layers of blanks can occur deformation affecting the quality of the treated surfaces. Such deformations have the greatest impact on the quality and accuracy of the details in the form of thin-walled cylindrical shells during their manufacture.

Known radial and axial residual stress in the circumferential surface layer of the thin-walled cylindrical work piece.

According to some researchers radial residual stress is 2 – 4 % of the hoop stress. Therefore, we assume that after machining the greatest impact on the quality of the treated surfaces have a flat or a biaxial stress state. According to research, and axial residual stresses are practically devoid of distortions of the shell and their action is not taken into account.

In developing the process of machining thin-walled parts is very important that the district diagrams such residual stresses obtained in which the value of deformation of the shell does not exceed the tolerance diametrical size of the shell. Character diagrams circumferential residual stress is determined by conditions of machining operations and depends largely on the unevenness of allowance, on uneven textures of the material, changing the shape of the grains and sizes, as well as disorders of the integrity of the surface layer material. On stress distribution are also affected by phase transformations occurring in the material of the surface layer over time.

These factors show that the surface layer of the work pieces by a significant localized residual stress value. For certain values of the stresses are uniformly distributed around the circumference, a thin-walled shell may lose its stable circular shape and move to another intermediate equilibrium. Since the residual stress field discrete, the resulting equilibrium state is maintained.

Studies have shown that the most characteristic deviations cross-sectional shapes are shell roundness, trihedral, tetrahedron, etc. (Fig. 1 a, b, c).

\[
\mathbf{J} = U_c + U_U - W
\]

when: \( U_c \) – the potential energy of deformation of the middle surface; \( U_U \) – potential energy of bending cross-section shell; \( W \) – the work of external forces (residual stress).

Potential energy of deformation of the middle surface \( U_c \) and bending of the cross-sectional \( U_U \) shell is determined by:

\[
U_c = \frac{h}{2E} \int \int [\nabla^4 \Phi -(1-\mu) L(\Phi, \Phi)] \, dx \, dy
\]

\[
U_U = \frac{E_1 h^3}{12 (1-\mu^2)} \int \int [(\nabla^2 \omega) -(1-\mu) L(\omega, \omega)] \, dx \, dy
\]
when: $h$ – the thickness of the shell wall; $E$ – modulus material; $\mu$ – Poisson’s ratio; $\nabla^2$ – Laplace operator; $L(\Phi, \Phi)$ – operator depending on the stress function; $L(\omega, \omega)$ – operator depending on the field of deflection; $x$, $y$ – coordinates along the length and circumference of the shell, respectively.

Work of external forces is given by:

$$W = q \iint_{\Omega} \omega \cdot dx \cdot dy$$  \hspace{1cm} (4)

when: $q$ – uniformly distributed transverse pressure equivalent by the action of the circumferential residual stresses.

To calculate the potential energy $U_c$ and $U_v$ shell using Laplace $\nabla^2$ and Laplace $\nabla^4$, acting in the formulas (2) and (3) in relation to the cylindrical shell.

Substituting these solutions, we define:

$$U_c = \frac{h}{2E} \left( \frac{L(\Phi, \Phi)}{0} + \frac{L(\omega, \omega)}{0} \right)^2 - 2(l + \mu) \left[ \frac{\partial^2 \Phi}{\partial x^2} \cdot \frac{\partial^2 \Phi}{\partial y^2} - \frac{\partial^2 \Phi}{\partial x \partial y} \right]dx\,dy$$ \hspace{1cm} (5)

$$U_v = \frac{Eh^2}{12(l - \mu)} \left( \frac{L(\Phi, \Phi)}{0} + \frac{L(\omega, \omega)}{0} \right)^2 - 2(l + \mu) \left[ \frac{\partial^2 \omega}{\partial x^2} \cdot \frac{\partial^2 \omega}{\partial y^2} - \frac{\partial^2 \omega}{\partial x \partial y} \right]dx\,dy$$ \hspace{1cm} (6)

To compute $U_c$ and $U_v$, necessary, based on the results of the study, ask mathematically approximating the expression for the deflection $\omega$.

Approximating expression for the deflection $\omega$ chosen in the form:

$$\omega = f \left( \sin \frac{ny}{R} + \sin^3 \frac{\pi x}{L} \right)$$ \hspace{1cm} (7)

In the particular case, $n = 2$ oval shape is obtained (Figure 1).

Determine the second derivatives $\frac{\partial^2 \omega}{\partial x^2}$ and $\frac{\partial^2 \omega}{\partial y^2}$ through the coordinates $X$, $Y$:

$$\frac{\partial^2 \omega}{\partial x^2} = \frac{1}{2} f \left( \frac{2\pi}{L} \right)^2 \cos \frac{2\pi x}{L}$$

$$\frac{\partial^2 \omega}{\partial y^2} = \left( \frac{n}{R} \right)^2 f \sin \frac{ny}{R}$$

$$\frac{\partial^2 \omega}{\partial x \partial y} = 0$$ \hspace{1cm} (8)

Determine the potential energy of the middle surface $U_c$. Stress function $\Phi$, the expression for $U_c$, we find, using the well-known equation strain compatibility.

Applied to cylindrical shells, this equation has the following form:

$$\frac{1}{E} \nabla^4 \Phi = - \frac{1}{2} L(\Phi, \Phi) - \frac{1}{R} \frac{\partial^2 \omega}{\partial x^2}$$ \hspace{1cm} (9)

Calculating the operator $L(\omega, \omega)$ find:

$$L(\omega, \omega) = - f^2 \frac{4\pi^2 n^2}{L^2 R^2} \sin \frac{ny}{R} \cos \frac{\pi x}{L}$$ \hspace{1cm} (10)

Substituting (8) and (10) into the right side of equation (9) we obtain:

$$\frac{1}{E} \nabla^4 \Phi = f^2 \frac{2\pi^2 n^2}{L^2 R^2} \sin \frac{ny}{R} \cos \frac{2\pi x}{L} - \frac{1}{R} \frac{L^2}{2\pi^2 R} \cos \frac{2\pi x}{L} - \frac{\sigma^*}{\pi} \frac{x^2}{2E}$$ \hspace{1cm} (11)

The integral of this expression:

$$\frac{1}{E} \Phi = f^2 A \cdot A, \sin \frac{ny}{R} \cos \frac{2\pi x}{L} - \frac{f^2 L^2}{8\pi^2 R} \cos \frac{2\pi x}{L} - \frac{\sigma^*}{\pi} \frac{x^2}{2E}$$ \hspace{1cm} (12)

Determine the partial derivatives of the stress on the $X$ and $Y$:

$$\frac{\partial^3 \Phi}{\partial x^3} = -EAA, f^2 \left( \frac{2\pi}{L} \right)^2 \sin \frac{ny}{R} \cos \frac{2\pi x}{L}$$

$$+ f \frac{L}{2R} \cos \frac{2\pi x}{L} - \frac{\sigma^*}{\pi}$$

$$\frac{\partial^3 \Phi}{\partial y^3} = -EAA, f^2 \left( \frac{n}{R} \right)^2 \sin \frac{ny}{R} \cos \frac{2\pi x}{L}$$

$$\frac{\partial^3 \Phi}{\partial x \partial y} = -EAA, f^2 \left( \frac{n}{R} \right)^2 \sin \frac{ny}{R} \cos \frac{2\pi x}{L}$$ \hspace{1cm} (13)

As a result, we find that $U_v = 0$.

This is explained by the fact that for the deflection $\omega$ selected expression that determines roundness without formation of local dents on the generator shell. In the case of distortion of the shell in a shallow roundness, we can assume the deformation energy of the medial surface is negligibly small compared with the energy of bending.
Calculate the potential energy of the shell cross-section bending $U_V$. To do this, the formula (8) we substitute the expression (6) for the energy $U_V$ and after the substitution of the limits of integration, we obtain:

$$U_V = \frac{Eh^3}{12(1-\mu^2)} f^3 \left( \frac{4\pi^4}{L^4} + \frac{n^4}{R^4} \right) \pi RL$$

(14)

Compute work residual stresses:

$$W = qf \int_0^{L_2} \left( \sin \frac{ny}{R} + \sin \frac{2\pi x}{L} \right) dx dy$$

(15)

By integrating and substituting limits, we find:

$$W = qf \pi RL$$

(16)

Between the external pressure $q$ and the residual hoop stresses $\sigma_\theta$ there is a dependence

$$\sigma_\theta = q \frac{R}{h}$$

(17)

After substituting (17) into (16), we obtain:

$$W = \sigma_\theta hfRL$$

(18)

The total energy of the shell is equal to:

$$\mathcal{E} = \frac{Eh^3}{12(1-\mu^2)} f^3 \left( \frac{4\pi^4}{L^4} + \frac{n^4}{R^4} \right) \pi RL - \sigma_\theta^* hfRL$$

(19)

Minimizing the total energy $\mathcal{E}$ by deflection $f$:

$$\frac{\partial \mathcal{E}}{\partial f} = 0$$

(20)

This expression (20) means that the shell will bend to such deflection $f$, whereby its potential energy is a minimum. It is known that at the minimum energy of all material is in an equilibrium state.

Applying this condition, we determine the dependence of the deflection $f$ of the district of residual stresses, shell size and physic-mechanical properties of the material of the work piece:

$$f' = \frac{6\sigma_\theta^*(1-\mu^2)h}{E\left[4(\pi R)^4/L^4 + n^4\right]} \left( \frac{R}{h} \right)^3$$

(21)

Consequently, the developed process work pieces type thin-walled cylindrical shells must ensure:

$$f' \leq TD$$

(22)

when: $f'$ – a deformation amount of the shell, which depends on the work piece material, the shell and the size of the residual stresses magnitude: $TD$ – diameter tolerance machined surface.

The results of the analysis and dimensioning of processing led to the following conclusions:

1. Accuracy and quality of processed cylindrical shells depends on the physical and mechanical properties of the material and dimensions of the shell occur in the surface layers of the material residual stresses.
2. Significantly reduce the resulting deformation of shells can be by creating stress, equal sign and magnitude.
3. These calculated curves allow the concept of developing a scientific foundation for finishing and stabilizing treatment technology of thin-walled parts.

**BIBLIOGRAPHY**


---

**Streszczenie**

Teoretyczne podstawy powiązań deformacji cienkich cylindrycznych powłok z działaniem naprężeń szczątkowych wzrastających po obróbce wiórowej. Wykorzystanie teorii stabilności energii powłok elastycznych.

**Słowa kluczowe**

naprężenia ściskające, naprężenia rozciągające, dokładność, tolerancje, pasowanie, wielkość łańcucha